MASTER LEVEL INTERNSHIP PROPOSAL:
ADIABATIC THEOREMS FOR MARKOV PROCESSES

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In this internship, we propose to the interested student to study sequences (X^n) of Markov processes on the time interval $[0,1]$ defined by stochastic differential equations such as

$$dX_t^n = nb(t, X_t^n)dt + \sqrt{n}\sigma(t, X_t^n)dW_t$$

with b and σ two functions from $[0,1] \times \mathbb{R}$ to $\mathbb{R}$. The goal is to characterize the limit $\lim_n X^n$ if it exists in any meaningful way. One hopes to prove the existence of such limit using a recent extension of adiabatic theorems from unitary groups to contracting semi groups[AFGG]. A first step will be to translate the assumptions of [AFGG] to this model. Said assumptions concern the family of Markov infinitesimal generators $(L_s)_{s\in[0,1]}$ defined by

$$L_s f(x) = b(s,x)\partial_x f(x) + \frac{1}{2}\sigma^2(s,x)\partial_x^2 f(x).$$

Assuming that for almost every $s \in [0,1]$ the Markov process generated by L_s accepts a unique invariant probability measure ν_s, the assumptions are formulated in terms of smoothness of $s \mapsto L_s$ and $s \mapsto \nu_s$. The smoothness of the invariant measure map requires the most work and may be first investigated for models with exponential mixing that can be studied through perturbation theory.

The limit we propose to investigate corresponds to a quasi static limit in a thermodynamic context. Hence, beyond the development of an adiabatic theory for Markov processes, one of the goal of the project attached to this internship is to rigorously prove quasi static thermodynamic laws, and deviations from them, from microscopic models.

Depending on the student interest, this project can then evolve into different directions, whether proving more general adiabatic theorems for Markov or non Markov processes, or delving into mathematical physics to study more refined thermodynamic properties such as linear response theory and large deviations for some relevant quantities such as entropy production or work. This work could eventually be continued during a Ph.D. Thesis in mathematical physics and probability.

References

[AFGG] Avron, J.E., Fraas, M., Graf, G.M., and Grech, P.: Adiabatic theorems for generators of contracting evolutions. Commun. Math. Phys. **314**, 163–191 (2012).

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